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Tidal torques dynamical friction and the structure of clusters of galaxies

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Abstract. We study the joint effect of tidal torques and dynamical friction on the collapse of density peaks solving numerically the equations of motion of a shell of barionic matter falling into the central regions of a cluster of galaxies. We calculate the evolution of the expansion parameter, $a(t)$, of the perturbation using a coefficient of dynamical friction η_{cl} obtained from a clustered system and taking into account the gravitational interaction of the quadrupole moment of the system with the tidal field of the matter of the neighboring proto-galaxies. We show that within high-density environments, such as rich clusters of galaxies, tidal torques and dynamical friction slow down the collapse of low-ν peaks producing an observable variation of the parameter of expansion of the shell. As a consequence a bias of dynamical nature arises because high-density peaks preferentially collapse to form halos within which visible objects eventually will condense. For a standard Cold Dark Matter model this *dynamical bias* can account for a substantial part of the total bias required by observations on cluster scales.

1.Introduction.

The origin and evolution of large scale structures is nowadays the outstanding problem in Cosmology. In a hierarchical 'bottom-up' scenario, high density collapsed peaks cluster

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and merge to forme larger structures. The density fluctuation field is often assumed to be locally isotropic, the amplitudes are Gaussian distributed with uncorrelated phases (Peebles 1980). Under these assumptions the power spectrum for a continous random field is basically the squared amplitude of its Fourier modes. If we assume, for computational convenience, that the random field $\delta(r)$ is periodic in some large rectangular volume V, we can define the Fourier trasform to be:

$$\delta(\mathbf{k}) = \frac{1}{V}\int \delta(\mathbf{r})\exp(i\mathbf{kr})d^3\mathbf{r} \tag{1}$$

According to Wiener-Khintchine theorem:

$$< |\delta_k|^2 >= \frac{1}{V}\int \xi(\mathbf{r})\exp(i\mathbf{kr})d^3\mathbf{r} \tag{2}$$

and

$$\xi(\mathbf{r}) = \frac{V}{2\pi^3}\int < |\delta_k|^2 > \exp(-i\mathbf{kr})d^3\mathbf{r} \tag{3}$$

i.e. the power spectrum, $P(k)$, is the Fourier trasform of the autocorrelation function and vice versa. Also the density fluctuation field, $\delta(r)$, can be obtained from the Fourier trasform of the power spectrum, $P(k)$. In other words, on average the characteristics of the density field peaks, e.g., their mass distribution, peculiar velocities, etc., are completely determined by the spectrum through its moments (at least during the linear and early non-linear phases of the collapse (Bardeen et al. 1986)). Moreover the isotropy condition imposes that all physical quantities around density peaks is, on average, spherically symmetric. However, actual realizations of, e.g., the density field distributions around the density peaks which eventually will give birth to galaxies and clusters, depart from spherical simmetry and from the average density profile, producing important consequences on collapse dynamics and formation of protostructures (Hoffman & Shaham 1985; Ryden 1988; Heavens & Peacock 1988; Kashlinsky 1986, 1987; Peebles 1990). A fundamental role in this context is played by the joint action of tidal torques (coupling shells of matter which are accreted around a density peak and neighboring protostructures (Ryden 1988)), and by dynamical friction (White 1976; Kashlinsky, 1986, 1987, Antonuccio & Colafrancesco 1995 (hereafter AC), Del Popolo & Gambera 1996). Some authors (see Barrow & Silk 1981, Szalay & Silk 1983, Peebles 1990) have proposed that non-radial motions would be expected within a developing proto-cluster, due to the tidal interaction of the irregular mass distribution around them, (typical of hierarchical clustering models), with the neighboring proto-clusters. The kinetic energy of this non-radial motions opposes the collapse of the proto-cluster, enabling the same to reach statistical equilibrium before the final collapse (the so called previrialization conjecture by Davis & Peebles

1977, Peebles 1990). Non-radial motions change the energetics of the collapse model by introducing another potential energy term. One expects that non-radial motions produce firstly a change in the turn around epoch, secondly a new functional form for δ_c, thirdly a change of the mass function calculable with the Press-Schechter (1974) formula and finally a modification of the two-point correlation function. Recently Colafrancesco, Antonuccio & Del Popolo (1995, hereafter CAD) have shown that dynamical friction delays the collapse of low-ν peaks inducing a bias of dynamical nature. Because of dynamical friction under-dense regions in clusters (the clusters outskirts) accrete less mass with respect to that accreted in absence of this dissipative effect and as a consequence over-dense regions are biased toward higher mass (Antonuccio & Colafrancesco 1995 and Del Popolo & Gambera, 1996). Dynamical friction and non-radial motions acts in a similar fashion: they delay the shell collapse consequently inducing a dynamical bias similar to that produced by dynamical friction but obviously of a larger value. This dynamical bias can be evaluated defining a selection function similar to that given in CAD and using Bardeen, Bond, Szalay and Kaiser (1986, hereafter BBKS) prescriptions.

The plan of the paper is the following: in §2 we obtain the total specific angular momentum acquired during expansion by a proto-cluster. In §3 we calculate the dynamical friction force for galaxies moving into the cluster, taking account of the clustering. In §4 we use the calculated specific angular momentum and the dynamical friction force to obtain the time of collapse of shells of matter around peaks of density having $\nu_c = 2, 3, 4$ and we compare the results with Gunn & Gott's (1972, hereafter GG) spherical collapse model. In §5 we derive a selection function for the peaks giving rise to proto-structures while in §6 we calculate some values for the bias parameter, using the selection function derived, on three relevant filtering scales. Finally in §7 we discuss the results obtained.

2. Tidal torques and angular momentum.

The explanation of galaxies spins gain through tidal torques was pioneered by Hoyle (1949). Peebles (1969) performed the first detailed calculation of the acquisition of angular momentum in the early stages of protogalactic evolution. More recent analytic computations (White 1984, Hoffman 1986, Ryden 1988a) and numerical simulations (Barnes & Efstathiou 1987) have re-investigated the role of tidal torques in originating galaxies angular momentum.

One way to study the variation of angular momentum with radius in a galaxy is that followed by Ryden (1988a). In this approach the protogalaxy is divided into a series of mass shells and the torque on each mass shell is computed separately. The density profile of each proto-structure is approximated by the superposition of a spherical profile, $\delta(r)$,

and a random CDM distribution, $\varepsilon(\mathbf{r})$, which provides the quadrupole moment of the protogalaxy. As shown by Ryden (1988a) the net rms torque on a mass shell centered on the origin of internal radius r and thickness δr is given by:

$$\langle|\tau|^2\rangle^{1/2} = \sqrt{30}\left(\frac{4\pi}{5}G\right)[\langle a_{2m}(r)^2\rangle\langle q_{2m}(r)^2\rangle - \langle a_{2m}(r)q^*_{2m}(r)\rangle^2]^{1/2} \tag{4}$$

where q_{lm}, the multipole moments of the shell and a_{lm}, the tidal moments, are given by:

$$\langle q_{2m}(r)^2\rangle = \frac{r^4}{(2\pi)^3}M^2_{sh}\int k^2 dk P(k)\, j_2(kr)^2 \tag{5}$$

$$\langle a_{2m}(r)^2\rangle = \frac{2\rho_b^2 r^{-2}}{\pi}\int dk P(k)\, j_1(kr)^2 \tag{6}$$

$$\langle a_{2m}(r)q^*_{2m}(r)\rangle = \frac{r}{2\pi^2}\rho_b M_{sh}\int k dk P(k)\, j_1(kr)\, j_2(kr) \tag{7}$$

where M_{sh} is the mass of the shell, $j_1(r)$ and $j_2(r)$ are the spherical Bessel function of first and second order while the power spectrum $P(k)$ is given by:

$$P(k) = Ak^{-1}[\ln(1+4.164k)]^2 (192.9 + 1340k + 1.599\times10^5k^2 + 1.78\times10^5k^3 + 3.995\times10^6k^4)^{-1/2} \tag{8}$$

(Ryden & Gunn 1987). The normalization constant A can be obtained, as usual, imposing that the mass variance at $8h^{-1}Mpc$, σ_8, is equal to unity. Filtering the spectrum on cluster scales, $R_f = 3h^{-1}Mpc$, we have obtained the rms torque, $\tau(r)$, on a mass shell using Eq. (4) then we obtained the total specific angular momentum, $h(r,\nu)$, acquired during expansion integrating the torque over time (Ryden 1988a Eq. 35):

$$h(r,\nu) = \frac{1}{3}\left(\frac{3}{4}\right)^{2/3} \frac{\tau_o t_0}{M_{sh}}\overline{\delta}_o^{-5/2}\int_0^\pi \frac{(1-\cos\theta)^3}{(\vartheta-\sin\vartheta)^{4/3}}\frac{f_2(\vartheta)}{f_1(\vartheta)-f_2(\vartheta)\frac{\delta_o}{\overline{\delta}_o}}d\vartheta \tag{9}$$

the functions $f_1(\vartheta)$, $f_2(\vartheta)$ are given by Ryden (1988a) (Eq. 31) while the mean over-density inside the shell, $\overline{\delta}(r)$, is given by Ryden (1988a):

$$\overline{\delta}(r,\nu) = \frac{3}{r^3}\int_0^\infty d\sigma\sigma^2\delta(\sigma) \tag{10}$$

In fig. 1 we show the variation of $h(r,\nu)$ with the distance r for three values of the peak height ν. The rms specific angular momentum, $h(r,\nu)$, increases with distance r while peaks of greater ν acquire less angular momentum via tidal torques. This is the angular

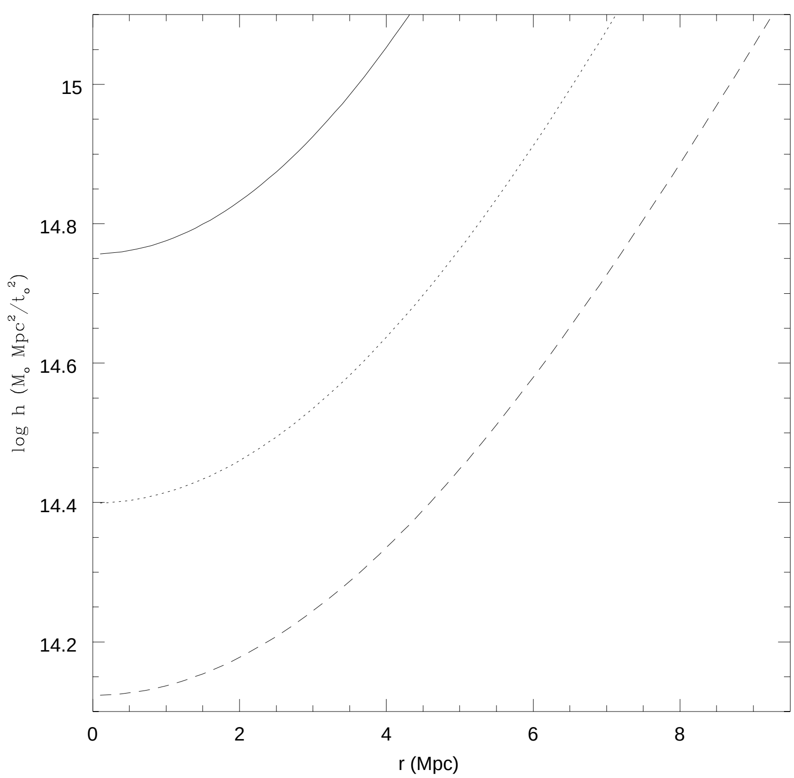

Fig. 1. The specific angular momentum, in units of $M_{\odot}$, Mpc and the Hubble time, t_o, for three values of the parameter ν ($\nu = 2$ solid line, $\nu = 3$ dotted line, $\nu = 4$ dashed line) and for $R_f = 3h^{-1}Mpc$.

momentum-density anticorrelation showed by Hoffman (1986). This effect arises because the angular momentum is proportional to the gain at turn around time, t_m, which in turn is proportional to $\overline{\delta}(r,\nu)^{-\frac{3}{2}} \propto \nu^{-3/2}$.

3. Dynamical friction.

In a hierarchical structure formation model, the large scale cosmic environment can be represented as a collisionless medium made of a hierarchy of density fluctuations whose mass, M, is given by the mass function $N(M,z)$, where z is the redshift. In these models matter is concentrated in lumps, and the lumps into groups and so on. In such a material system, gravitational field can be decomposed into an average field, $\mathbf{F}_0(r)$, generated from the smoothed out distribution of mass, and a stochastic component, $\mathbf{F}_{stoch}(r)$, generated from the fluctuations in number of the field particles. The stochastic component of the gravitational field is specified assigning a probability density, $W(\mathbf{F})$, (Chandrasekhar & von Neumann 1942). In an infinite homogeneous unclustered system $W(\mathbf{F})$ is given by Holtsmark distribution (Chandrasekhar & von Neumann 1942) while in

inhomogeneous and clustered systems $W(\mathbf{F})$ is given by Kandrup (1980) and Antonuccio-Delogu & Barandela (1992) respectively. The stochastic force, $\mathbf{F}_{stoch}$, in a self-gravitating system modifies the motion of particles as it is done by a frictional force. In fact a particle moving faster than its neighbours produces a deflection of their orbits in such a way that average density is greater in the direction opposite to that of traveling causing a slowing down in its motion.

Following Chandrasekhar & von Neumann's (1942) method, the frictional force which is experienced by a body of mass M (galaxy), moving through a homogeneous and isotropic distribution of lighter particles of mass m (substructure), having a velocity distribution $n(v)$ is given by:

$$M\frac{d\mathbf{v}}{dt} = -4\pi G^2 M^2 n(v)\frac{\mathbf{v}}{v^3}\log\Lambda\rho \tag{11}$$

where $\log\Lambda$ is the Coulomb logarithm, ρ the density of the field particles (substructure). A more general formula is that given by Kandrup(1980) in the hypothesis that there are no correlations among random force and their derivatives:

$$\mathbf{F} = -\eta\mathbf{v} = -\frac{\int W(F)F^2T(F)d^3F}{2<v^2>}\mathbf{v} \tag{12}$$

where η is the coefficient of dynamical friction, $T(F)$ the average duration of a random force impulse, $<v^2>$ the characteristic speed of a field particle having a distance $r \simeq (\frac{GM}{F})^{1/2}$ from a test particle (galaxy). This formula is more general than Eq. (11) because the frictional force can be calculated also for inhomogeneous systems when $W(F)$ is given. If the field particles are distributed homogeneously the dynamical friction force is given by:

$$F = -\eta v = -\frac{4.44 G^2 m_a^2 n_a}{[<v^2>]^{3/2}}\log\left\{1.12\frac{<v^2>}{G m_a n_a^{1/3}}\right\} \tag{13}$$

(Kandrup 1980), where m_a and n_a are respectively the average mass and density of the field particles. Using virial theorem we also have:

$$\frac{<v^2>}{G m_a n_a^{1/3}} \simeq \frac{M_{tot}}{m}\frac{1}{n^{1/3}R_{sys}} \simeq N^{2/3} \tag{14}$$

where M_{tot} is the total mass of the system, R_{sys} its radius and N is the total number of field particles. The dynamical friction force can be written as follows:

$$F = -\eta v = -\frac{4.44[G m_a n_{ac}]^{1/2}}{N}\log\left\{1.12N^{2/3}\right\}\frac{v}{a^{3/2}} \tag{15}$$

where $N = \frac{4\pi}{3}R_{sys}^3 n_a$ and $n_{ac} = n_a \times a^3$ is the comoving number density of peaks of substructure of field particles. This last equation supposes that the field particles generating the stochastic field are virialized. This is justified by the previrialization hypothesis

(Davis & Peebles 1977).

To calculate the dynamical evolution of the galactic component of the cluster it is necessary to calculate the number and average mass of the field particles generating the stochastic field.

The protocluster, before the ultimate collapse at $z \simeq 0.02$, is made of substructure having masses ranging from $10^6 - 10^9 M_{\odot}$ and from galaxies. We suppose that the stochastic gravitational field is generated from that portion of substructure having a central height ν larger than a critical threshold ν_c. This latter quantity can be calculated (following AC) using the condition that the peak radius, $r_{pk}(\nu \geq \nu_c)$, is much less than the average peak separation $n_a(\nu \geq \nu_c)^{-1/3}$, where n_a is given by the formula of BBKS for the upcrossing points:

$$n_{ac}(\nu \geq \nu_c) = \frac{\exp(\nu_c^2/2)}{(2\pi)^2}(\frac{\gamma}{R_*})^3[\nu_c^2 - 1 + \frac{4\sqrt{3}}{5\gamma^2(1-5\gamma^2/9)^{1/2}}\exp(-5\gamma^2\nu_c^2/18)] \tag{16}$$

where γ, R_* are parameters related to moments of the power spectrum (BBKS Eq. 4.6A). The condition $r_{pk}(\nu \geq \nu_c) < 0.1 n_a(\nu \geq \nu_c)^{-1/3}$ ensures that the peaks of substructure are point like. Using the radius for a peak:

$$r_{pk} = \sqrt{2} R_* \left[\frac{1}{(1+\nu\sigma_0)(\gamma^3 + (0.9/\nu))^{3/2}}\right]^{1/3} \tag{17}$$

(AC), we obtain a value of $\nu_c = 1.3$ and then we have $n_a(\nu \geq \nu_c) = 50.7 Mpc^{-3}$ ($\gamma = 0.4$, $R_* = 50 Kpc$) and m_a is given by:

$$m_a = \frac{1}{n_a(\nu \geq \nu_c)} \int_{\nu_c}^{\infty} m_{pk}(\nu) N_{pk}(\nu) d\nu = 10^9 M_{\odot} \tag{18}$$

(in accordance with the result of AC), where m_{pk} is given in Peacock & Heavens (1990) and N_{pk} is the average number density of peak (BBKS Eq. 4.4). Clusters of galaxies are correlated systems whose autocorrelation function can be expressed, in the range $10 h^{-1} Mpc < r < 60 h^{-1} Mpc$, in a power law form:

$$\xi_{cc} = (\frac{r_{o,c}}{r})^{\gamma} \tag{19}$$

with $\gamma \simeq 1.8$ and a correlation length, $r_{o,c} \simeq 25 h^{-1} Mpc$ (Bahcal & Soneira 1983; Postman et al. 1986). The analysis of fair samples of galaxies gives for the galaxy autocorrelation function the expression:

$$\xi_{gg} = (\frac{r_{o,g}}{r})^{\gamma} \tag{20}$$

in the range $0.1h^{-1}Mpc < r < 10h^{-1}Mpc$ ($r_{o,g} \simeq 5h^{-1}Mpc$, $\gamma = 1.77 \pm 0.03$ (Peebles 1980, Davis & Peebles 1983)). The description of dynamical friction in these systems need to use a distribution of the stochastic forces, $W(F)$, taking account of correlations. In this last case the coefficient of dynamical friction, η, may be calculated using the equation:

$$\eta = \int d^3\mathbf{F} W(F) F^2 T(F)/(2 < v^2 >) \tag{21}$$

and using Antonuccio & Atrio (1992) distribution:

$$W(F) = \frac{1}{2\pi^2 F} \int_0^\infty dkksin(kF) A_f(k) \tag{22}$$

where A_f is given in the quoted paper. The function A_f is a linear integral function of the correlation function $\xi(r)$. As shown in Del Popolo & Gambera (1997) the effect of clustering is that to increase the effects of dynamical friction.

4. Shell collapse time. Tidal torques and dynamical friction acts in a similar fashion. As known one of the consequences of the angular momentum acquisition by a mass shell of a proto-cluster is the delay of the collapse of the proto-structure. As shown by Barrow & Silk (1981) and Szalay & Silk (1983) the gravitational interaction of the irregular mass distribution of proto-cluster with the neighbouring proto-structures gives rise to non-radial motions, within the protocluster, which are expected to slow the rate of growth of the density contrast and to delay or suppress collapse. According to Davis & Peebles (1977) the kinetic energy of the resulting non-radial motions at the epoch of maximum expansion increases so much to oppose the recollapse of the proto-structure. Numerical N-body simulations by Villumsen & Davis (1986) showed a tendency to reproduce this so called previrialization effect. In a more recent paper by Peebles (1990) the slowing of the growth of density fluctuations and the collapse suppression after the epoch of the maximum expansion were re-obtained using a numerical action method. In the central regions of a density peak ($r \leq 0.5R_f$) the velocity dispersion attain nearly the same value (Antonuccio & Colafrancesco 1997) while at larger radii ($r \geq R_f$) the radial component is lower than the tangential component. This means that motions in the outer regions are predominantly non-radial and in these regions the fate of the infalling material could be influenced by the amount of tangential velocity relative to the radial one. This can be shown writing the equation of motion of a spherically symmetric mass distribution with density $n(r)$ (Peebles 1993):

$$\frac{\partial}{\partial t} n\langle v_r \rangle + \frac{\partial}{\partial r} n\langle v_r^2 \rangle + \left(2\langle v_r^2 \rangle - \langle v_\vartheta^2 \rangle\right) \frac{n}{r} + n(r) \frac{\partial}{\partial t} \langle v_r \rangle = 0 \tag{23}$$

where $\langle v_r \rangle$ and $\langle v_\vartheta \rangle$ are, respectively, the mean radial and tangential streaming velocity. Eq. (23) shows that high tangential velocity dispersion ($\langle v_\vartheta^2 \rangle \geq 2\langle v_r^2 \rangle$) may alter the infall

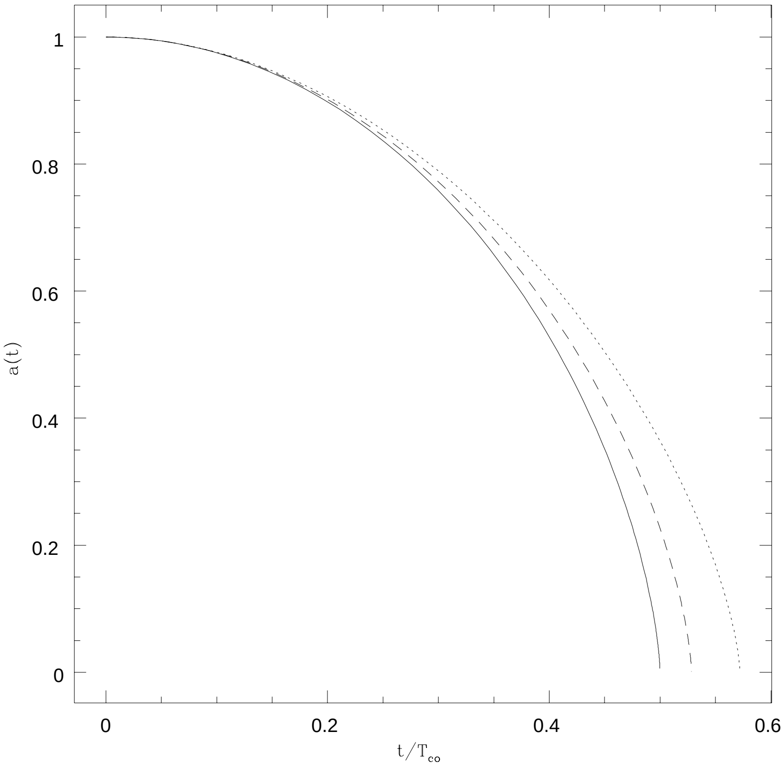

Fig. 2. The time evolution of the expansion parameter. The solid line is a(t) for GG model; the dashed line is a(t) taking account only dynamicalfriction; the dotted line is a(t) taking account of the cumulative effect of non-radial motions and dynamical friction.

pattern. The expected delay in the collapse of a perturbation, due to non-radial motions and dynamical friction, may be calculated solving the equation for the radial acceleration (Kashlinsky 1986, 1987; AC; Peebles 1993):

$$\frac{dv_r}{dt} = \frac{L^2(r,\nu)}{M^2 r^3} - g(r) - \eta \frac{dr}{dt} \tag{24}$$

where $L(r,\nu)$ is the angular momentum and $g(r)$ the acceleration. Writing the proper radius of a shell in terms of the expansion parameter, $a(r_i,t)$:

$$r(r_i,t) = r_i a(r_i,t) \tag{25}$$

remembering that

$$M = \frac{4\pi}{3}\rho_b(r_i,t)a^3(r_i,t)r_i^3 \tag{26}$$

and that $\rho_b = \frac{3H_0^2}{8\pi G}$, where H_0 is the Hubble constant and assuming that no shell crossing occurs so that the total mass inside each shell remains constant, that is:

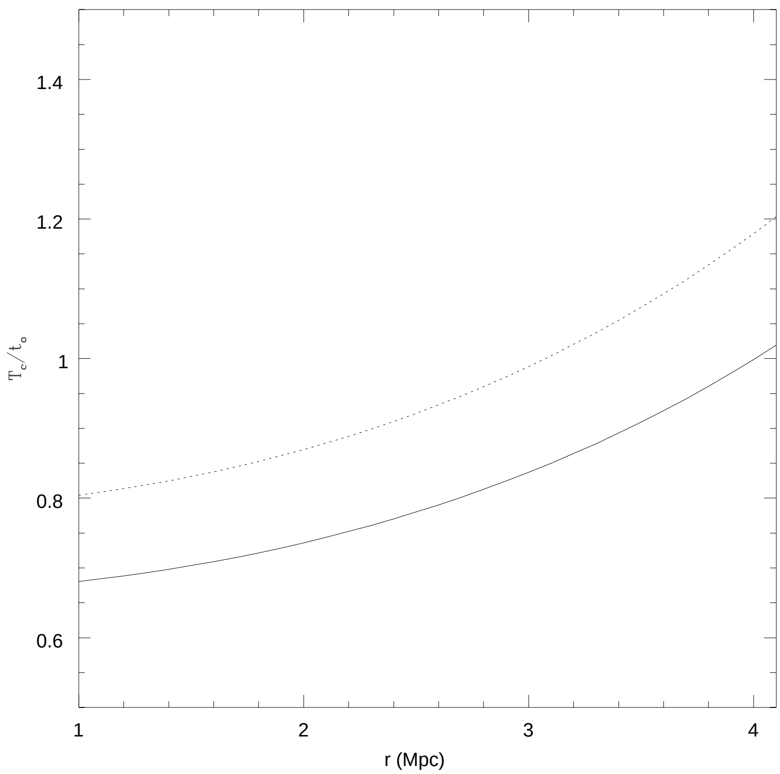

Fig. 3. The time of collapse of a shell of matter in units of the age of the universe t_o for $\nu = 2$ (dotted line) compared with Gunn & Gott's model (solid line).

$$\rho(r_i,t) = \frac{\rho_i(r_i,t)}{a^3(r_i,t)} \tag{27}$$

the Eq. (24) may be written as:

$$\frac{d^2a}{dt^2} = -\frac{H^2(1+\overline{\delta})}{2a^2} + \frac{4G^2L^2}{H^4(1+\overline{\delta})^2 r_i^{10} a^3} - \eta\frac{da}{dt} \tag{28}$$

The equation (28) may be solved using the initial conditions: $(\frac{da}{dt}) = 0$, $a = a_{max} \simeq 1/\overline{\delta}$ and using the function $h(r,\nu) = L(r,\nu)/M_{sh}$ found in §2 to obtain a(t) and the time of collapse, $T_c(r,\nu)$.

In Fig. 2 we show the effects of non-radial motions and dynamical friction separately. As displayed non-radial motions have a larger effect on the collapse delay with respect to dynamical friction. In Figs. 3 ÷ 5 we compare the results for the time of collapse, T_c, for $\nu = 2,3,4$ with the time of collapse of the classical GG spherical model:

$$T_{c0}(r,\nu) = \frac{\pi}{H_i}[\overline{\delta}(r,\nu)]^{-3/2} \tag{29}$$

As shown the presence of non-radial motions produces an increase in the time of collapse of a spherical shell. The collapse delay is larger for low value of ν and becomes negligible

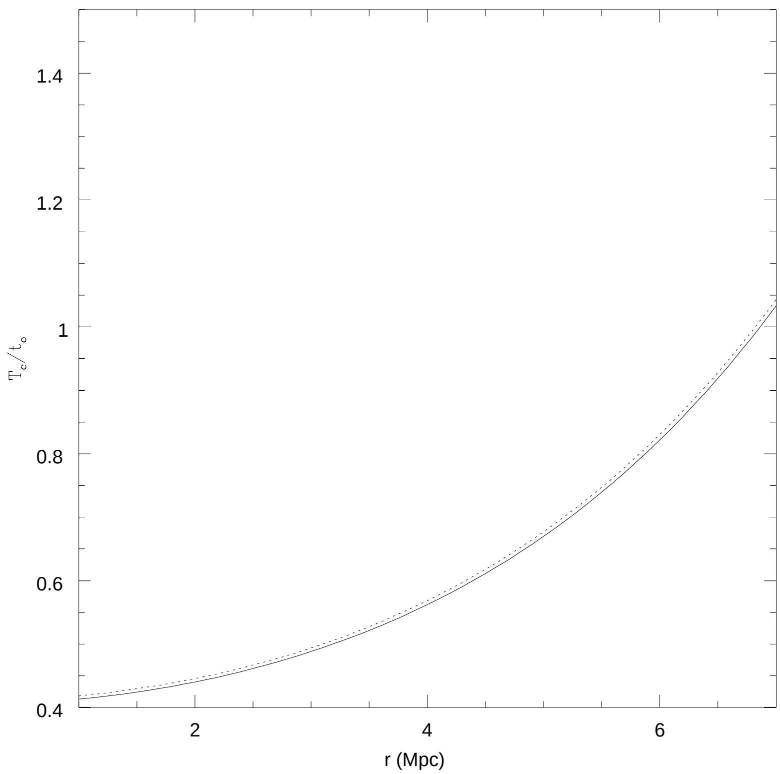

Fig. 4. The time of collapse of a shell of matter in units of the age of the universe t_o for $\nu = 3$ (dotted line) compared with Gunn & Gott's model (solid line).

for $\nu \geq 3$. This result is in agreement with the angular momentum-density anticorrelation effect: density peaks having low value of ν acquire a larger angular momentum than high ν peaks and consequently the collapse is more delayed with respect to high ν peaks. Given $T_c(r, \nu)$ we also calculated the total mass gravitationally bound to the final non-linear configuration. There are at least two criteria to establish the bound region to a perturbation $\delta(r)$: a statistical one (Ryden 1988b), and a dynamical one (Hoffman & Shaham 1985). The dynamical criterion, that we have used, supposes that the binding radius is given by the condition that a mass shell collapse in a time, T_c, smaller than the age of the universe t_0:

$$T_c(r, \nu) \leq t_0 \tag{30}$$

We calculated the time of collapse of GG spherical model, $T_{c0}(r, \nu)$, using the density profiles given in Ryden & Gunn (1987) for $1.7 < \nu < 4$ and then repeated the calculation taking into account non-radial motions obtaining $T_c(r, \nu)$. Then we calculated the binding radius, $r_{bo}(\nu)$, for a GG model solving $T_{c0}(r, \nu) \leq t_0$ for r and for several value of ν, while

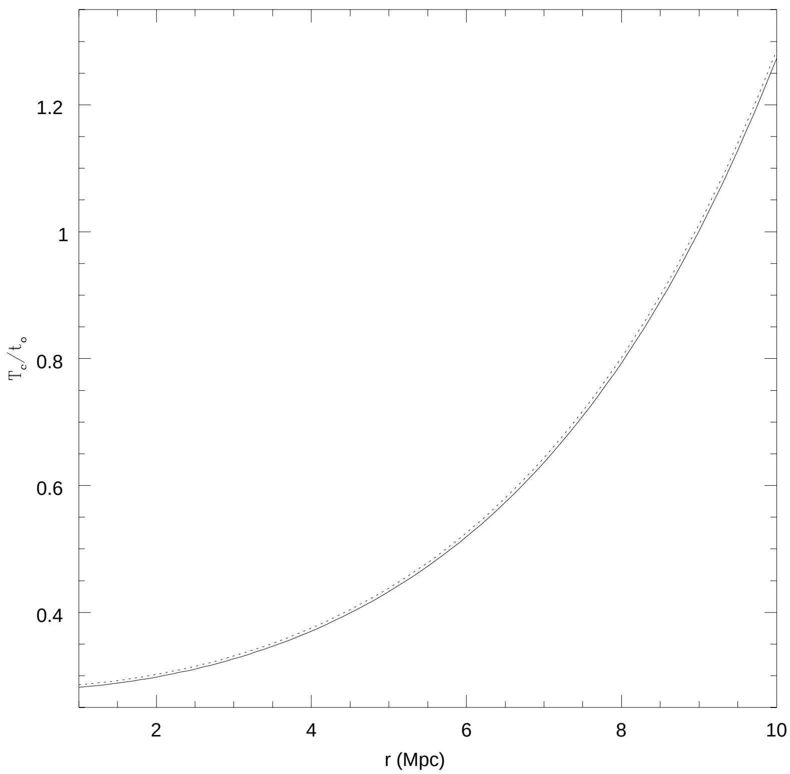

Fig. 5. The time of collapse of a shell of matter in units of the age of the universe t_o for $\nu = 4$ (dotted line) compared with Gunn & Gott's model (solid line).

we calculated the binding radius of the model that takes into account non-radial motions, $r_b(\nu)$, repeating the calculation, this time with $T_c(r, \nu) \leq t_0$. We found a relation between ν and the mass of the cluster using the equation: $M = \frac{4\pi}{3} r_b^3 \rho_b$.
In fig. 6 we compare the peak mass obtained from GG model, using Hoffman & Shaham's (1985) criterion, with that obtained from the model taking into account non-radial motions. As shown for high values of ν ($\nu \geq 3$) the two models give the same result for the mass while for $\nu < 3$ the effect of non-radial motions produces less bound mass with respect to GG model. decreases the effect of non-radial motions produces a decrease in the bound mass.

5. Tidal field and the selection function

Following BBKS we define a selection function $t(\nu/\nu_t)$ which gives the probability that a density peak forms an object, while the threshold level, ν_t, is defined so that the probability that a peak form an object is 1/2 when $\nu = \nu_t$. The selection function introduced by BBKS (Eq. 4.13), is an empirical one

and depends on two parameters: the threshold ν_t and the shape parameter q:

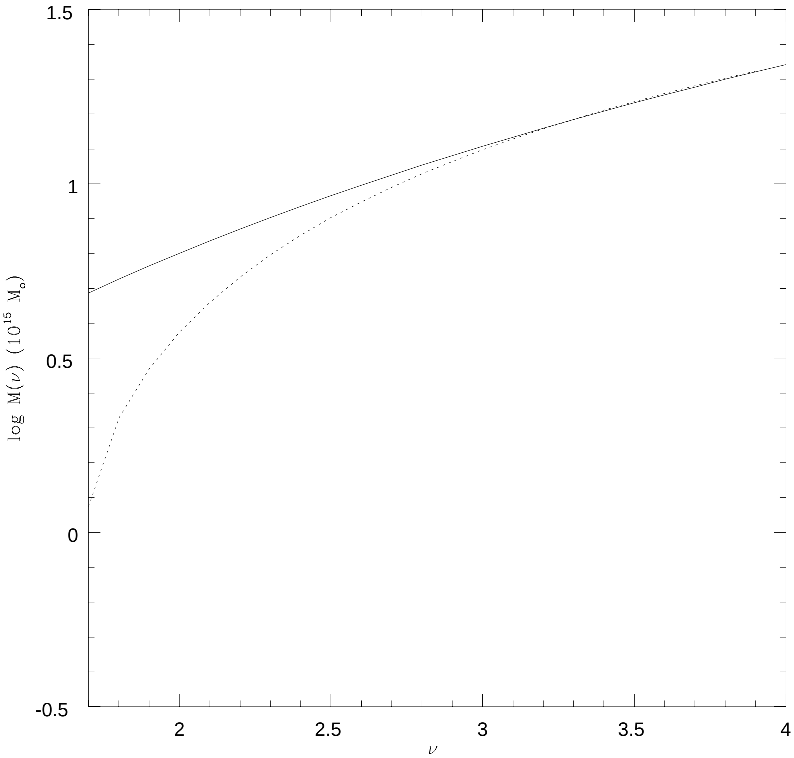

Fig. 6. The mass accreted by a collapsed perturbation, in units of $10^{15} M_\odot$, taking into account non-radial motions and dynamical friction effect (dotted line) compared to Gunn & Gott's mass (solid line).

$$t(\nu/\nu_t) = \frac{(\nu/\nu_t)^q}{1+(\nu/\nu_t)^q} \tag{31}$$

If $q \rightarrow \infty$ this selection function is a Heaviside function $\vartheta(\nu-\nu_t)$ so that peaks with $\nu > \nu_t$ have a probability equal to 100% to form objects while peaks with $\nu \leq \nu_t$ do not form objects. If q has a finite value sub-ν_t peaks are selected with non-zero probability. Using the given selection function the cumulative number density of peaks higher than ν is given, according to BBKS, by:

$$n_{pk} = \int_{\nu}^{\infty} t(\nu/\nu_t) N_{pk}(\nu) d\nu \tag{32}$$

where $N_{pk}(\nu)$ is the comoving peak density (see BBKS Eq. 4.3). A form of the selection function, physically motivated, can be obtained following the argument given in CAD. In this last paper the selection function is defined as:

$$t(\nu) = \int_{\delta_c}^{\infty} p\left[\overline{\delta}, \langle\overline{\delta}\rangle(r_{Mt},\nu), \sigma_{\overline{\delta}}(r_{Mt},\nu)\right] d\delta \tag{33}$$

where the function

$$p\left[\overline{\delta}, \langle\overline{\delta}\rangle(r)\right] = \frac{1}{\sqrt{2\pi}\sigma_{\overline{\delta}}} \exp\left(-\frac{|\overline{\delta} - \langle\overline{\delta}\rangle(r)|^2}{2\sigma^2_{\overline{\delta}}}\right) \tag{34}$$

gives the probability that the peak overdensity is different from the average, in a Gaussian density field. The selection function depends on ν through the dependence of $\overline{\delta}(r)$ from ν. As displayed the integrand is evaluated at a radius r_{Mt} which is the typical radius of the object we are selecting. Moreover the selection function $t(\nu)$ depends on the critical overdensity threshold for the collapse, δ_c, which is not constant as in a spherical model (due to the presence, in our analysis, of non-radial motions and dynamical friction that delay the collapse of the proto-cluster) but it depends on ν. An analityc determination of $\delta_c(\nu)$ can be obtained following a technique similar to that used by Bartlett & Silk (1993). Using Eq. (28) it is possible to obtain the value of the expansion parameter of the turn around epoch, a_{max}, which is characterized by the condition $\frac{da}{dt} = 0$. Using the relation between v and δ_i, in linear theory (Peebles 1980), we find

$$\delta_c(\nu) = \delta_{co}\left[1 + \frac{\lambda_o}{1 - \mu(\delta)} + \frac{8G^2}{\Omega_o^3 H_o^6 r_i^{10} \overline{\delta}(1+\overline{\delta})^2} \int \frac{L^2 da}{a^3}\right] \tag{35}$$

where $\delta_{co} = 1.68$ is the critical threshold for GG model and λ_o and $\mu(\delta)$ are given in Colafrancesco, Antonuccio & Del Popolo (1995) (Eq. 5, 6). In Fig. 7 we show the overdensity threshold in function of ν. As shown, $\delta_c(\nu)$ decreases with increasing ν. When $\nu > 3$ the threshold assume the typical value of the spherical model. This means, according to the cooperative galaxy formation theory, (Bower et al. 1993) that structures form more easily if there are other structures nearby, i.e. the threshold level is a decreasing function of the mean mass density. Known $\delta_c(\nu)$ and chosen a spectrum, the selection function is immediately obtainable through Eq. (33) and Eq. (34). The result of the calculation, plotted in fig. 8, for two values of the filtering radius, ($R_f = 2,\ 3\ h^{-1}Mpc$), shows that the selection function, as expected, differs from an Heaviside function (sharp threshold). The value of ν at which the selection function reaches the value 1 ($t(\nu) \simeq 1$) increases for increasing values of the filtering radius, R_f. This is due to the smoothing effect of the filtering process. The effect of non-radial motions is, firstly, that of shifting $t(\nu)$ towards higher values of ν, and, secondly, that of making it steeper. The selection function is also different from that used by BBKS (tab. 3a). Finally it is interesting to note that the selection function defined by Eq. (33) and Eq. (34) is totally general, it does not depend on the presence or absence of non-radial motions. The latter influence the selection function form through the changement of δ_c induced by non-radial motions itself.

6. The bias coefficient One way of defining the bias coefficient of a class of objects is that given by (BBKS):

$$b(R_f) = \frac{< \widetilde{\nu} >}{\sigma_o} + 1 \qquad (36)$$

where $\langle \widetilde{\nu} \rangle$ is:

$$< \widetilde{\nu} > = \int_0^\infty \left[\nu - \frac{\gamma\theta}{1-\gamma^2} \right] t(\frac{\nu}{\nu_t}) N_{pk}(\nu) d\nu \qquad (37)$$

from Eq. (37) it is clear that the bias parameter can be calculated once a spectrum, $P(k)$, is fixed. The bias parameter depends on the shape and normalization of the power spectrum. A larger value is obtained for spectra with more power on large scale (Kauffmann et al. 1996). In this calculation we continue to use the standard CDM spectrum ($\Omega_0 = 1$, $h = 0.5$) normalized imposing that the rms density fluctuations in a sphere of radius $8h^{-1}Mpc$ is the same as that observed in galaxy counts, i.e. $\sigma_8 = \sigma(8h^{-1}Mpc) = 1$. The calculations have been performed for three different values of the filtering radius ($R_f = 2$, 3, 4 $h^{-1}Mpc$). The result of the calculation is plotted in table 1. As shown, the value of the bias parameter tends to increase with R_f due the filter effect of $t(\nu)$. As shown $t(\nu)$ acts as a filter, increasing the filtering radius, R_f, the value of ν at which $t(\nu) \simeq 1$ increases . In other words when R_f increases $t(\nu)$ selects density peaks of larger height. The reason of this behavior must be searched in the smoothing effect that the increasing of the filtering radius produces on density peaks. When R_f is increased the density field is smoothed and $t(\nu)$ has to shift towards higher value of ν in order to select a class of object of fixed mass, M.

Bias

$R_f(h^{-1}Mpc)$	b
2	1.6
3	1.93
4	2.25

Table 1. Values of the coefficient of bias for different values of the filtering radius.

7. Conclusions

In this paper we have studied the role of non-radial motions and dynamical friction on the collapse of density peaks solving numerically the equations of motion of a shell of barionic matter falling into the central regions of a cluster of galaxies. We have shown that non-radial motions and dynamical friction produce a delay in the collapse of density

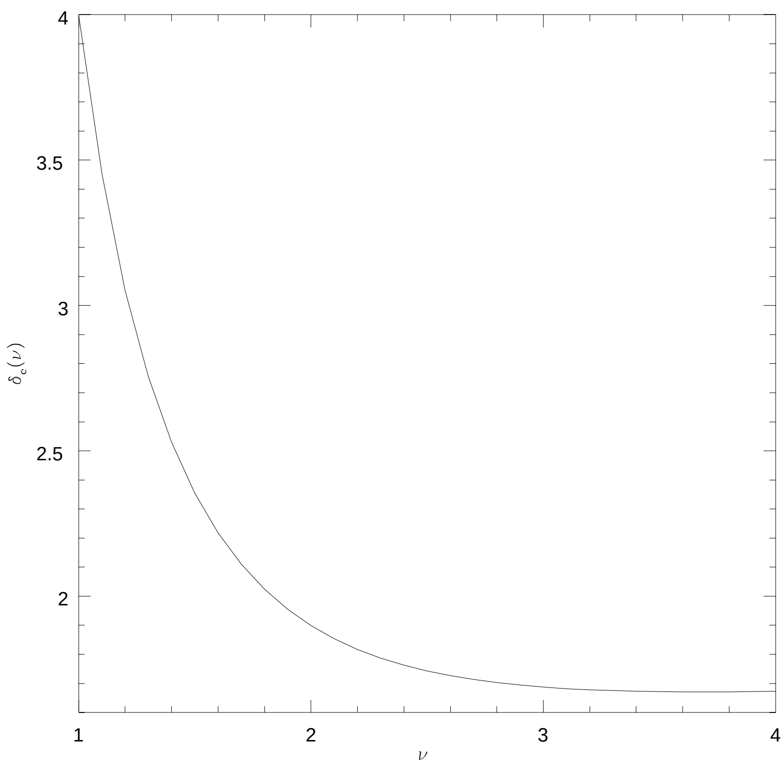

Fig. 7. The critical threshold, $\delta_c(\nu)$ versus ν

peaks having low value of ν while the collapse of density peaks having $\nu > 3$ is not influenced. A first consequence of this effect is a reduction of the mass bound to collapsed perturbations and a raising of the critical threshold, δ_c, which now is larger than that of the top-hat spherical model and depends on ν. This means that shells of matter of low density have to be subjected to a larger gravitational potential, with respect to the homogeneous GG model, in order to collapse. The delay in the proto-structures collapse gives rise to a dynamical bias similar to that described in CAD whose bias parameter may be obtained once a proper selection function is defined. The selection function found is not a pure Heaviside function and is different from that used by BBKS to study the statistical properties of clusters of galaxies. Its shape depends on the effect of non-radial motions and dynamical friction through its dependence on $\delta_c(\nu)$. The function $t(\nu)$ selects density peaks higher and higher with increasing value of R_f due to the smoothing effect of the density field produced by the filtering procedure. Using this selection function and BBKS prescriptions we have calculated the coefficient of bias, b. On clusters scales for $R_f = 4h^{-1}Mpc$ we found a value of $b = 2.25$ comparable with that obtained from the mean mass-to-light ratio of clusters, APM survey, or from N-body simulations combined

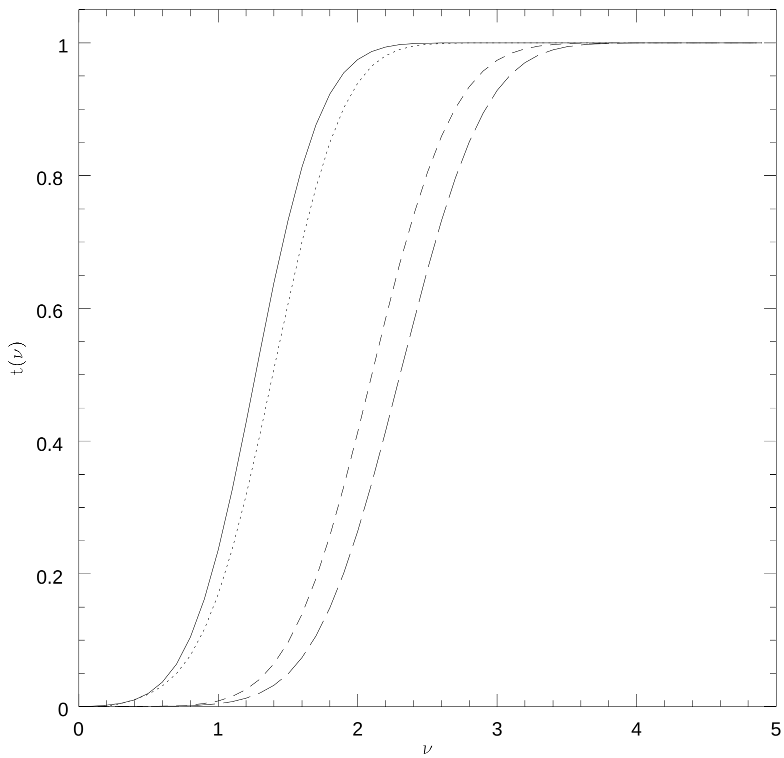

Fig. 8. The selection function, $t(\nu)$, for $R_f = 3h^{-1}Mpc$ ($\delta_c = 1.68$, solid line; δ_c function of ν, dotted line) and for $4h^{-1}Mpc$ ($\delta_c = 1.68$, short dashed line; δ_c function of ν, long dashed line).

with hydrodynamical models (Frenk et al. 1990). Besides, the value of the coefficient of biasing b that we have calculated is comparable with the values of b given by Kauffmann et al. 1996. This means that non-radial motions and dynamical friction play a significant role in determining the bias level.

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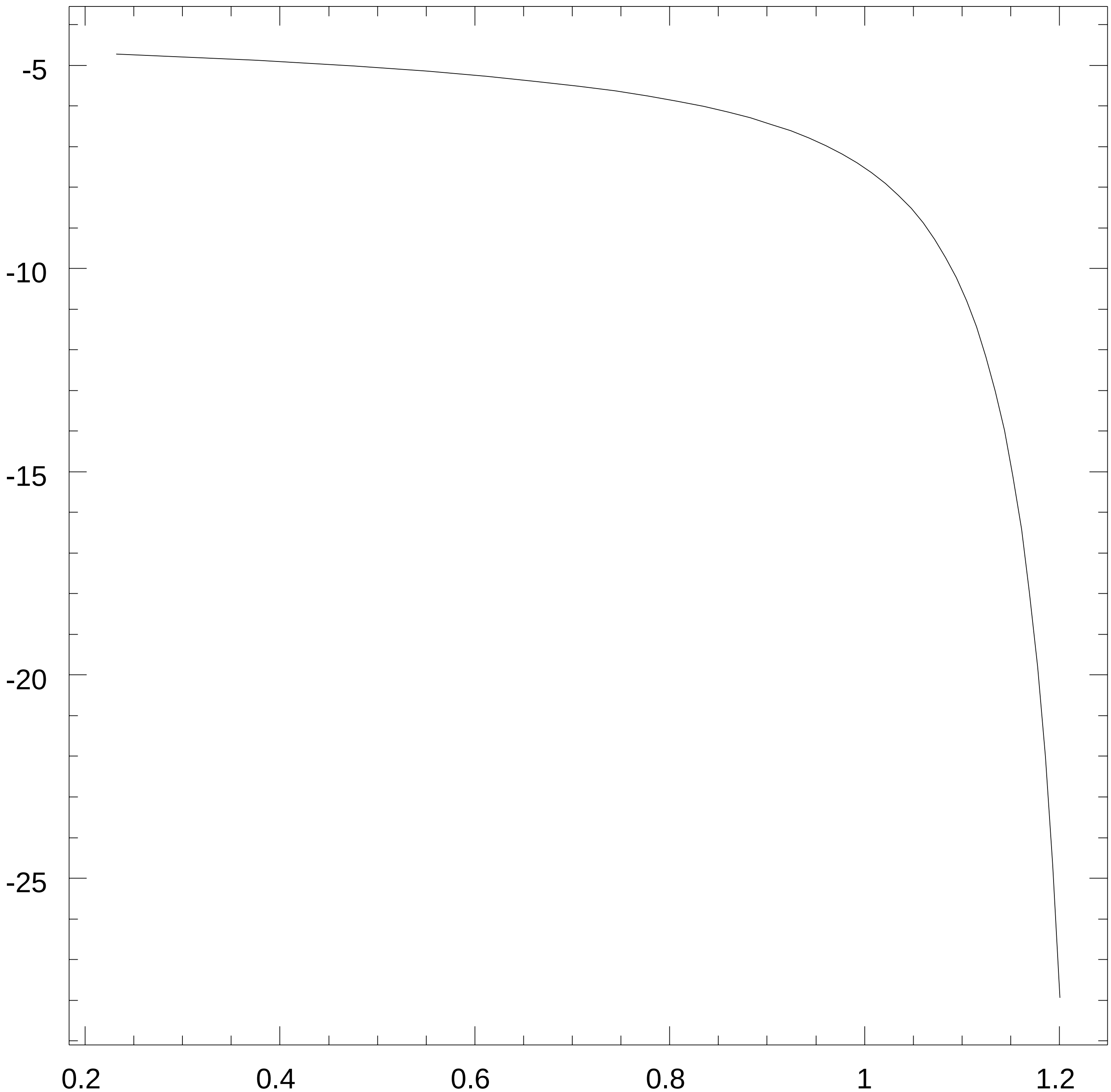
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